

A topology control protocol based on eligibility and efficiency metrics

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ABSTRACT

The question of fairness in wireless sensor networks is not studied very well. It is not unusual to observe in the literature fairness traded for low latency or reliability. However, a disproportional use of some critical nodes as relaying nodes can cause premature network fragmentation. This paper investigates fairness in multi-hop wireless sensor networks and proposes a topology control protocol that enables nodes to exhaust their energy fairly. Moreover, it demonstrates that whereas the number of neighboring nodes with which a node should cooperate depends on the density of the network, increasing this number beyond a certain amount does not contribute to network connectivity.

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1. Introduction

In wireless sensor networks, communication (receiving as well as transmitting) consumes a significant amount of energy. Since routing involves several nodes, its energy cost outweighs the cost of data processing. As to the exact number of nodes that should participate in a routing task, so far the research community is not in agreement. There are those who argue that multi-hop communication is preferred over single hop communication. One of the premises for this assumption is that as the distance of communication increases, the probability of getting a line-of-sight (LOS) link decreases, in which case the path loss index can no longer be assumed to be 2 but between 2 and 4, and in some cases, even 6. By reducing the distance of communication to a shorter length, it is possible to keep a LOS link, which significantly reduces the transmission cost.

On the other hand, there are those (for example Ephremides, 2002; Haenggi, 2004) who argue that this is an oversimplified analysis that does not take into account the cost of routing overhead, delay, channel coding/decoding, end-to-end reliability, efficiency of transmission power amplifiers, etc., and advocate long-hop routing. For densely and randomly deployed wireless sensor networks (such as in pipelines with several turns in short distances), short-hop routing is quite unsuitable. Apparently, long distance communication has also its disadvantages besides path loss, including interference.

A topology control protocol is necessary to set an upper and lower bound on the number of links that can be active in the

network. This ensures that the network remains connected and its lifetime is optimized. Moreover, it guarantees an available link to a higher level routing protocol that is defined based on an application-specific metric (such as minimum hop, minimum delay, minimum energy consumption, and maximum available power). Fig. 1 displays how a topology control protocol can be employed to trim off inefficient links in a wireless (sensor) network.

In wired networks, the way the network elements are physically interconnected directly influences the network's topology. Routing protocols take this fact into account when routes are computed. In wireless networks, however, as long as the communication range suffices, essentially all nodes can establish a link with each other, creating a mesh-topology network (Ephremides, 2002), which is not energy efficient. Another problem is that during the operation of the network, some nodes may exhaust their energy more rapidly than others while others may become dysfunctional. A topology control protocol deals with all these dynamics and ensures that the network is connected with energy-efficient links.

The main challenge is to develop a topology control strategy that is simple, scalable, and less resource intensive. Ideally, it should function based on local information only. In most cases, additional knowledge such as the placement and relative position of a node to the sink node can be obtained from layout information or from blueprints; and can be used to determine relative neighborhood. We propose a localized algorithm that enables nodes to autonomously create and maintain energy-efficient links. The protocol defines proximity and eligibility metrics to ensure network connectivity and to optimize lifetime.

The paper is organized as follows: First, we discuss related work in Section 2. In Section 3, the network model and the basic assumptions of the model are outlined. In Section 4, the theoretical concept of the fair and efficient topology control (FETC) protocol is

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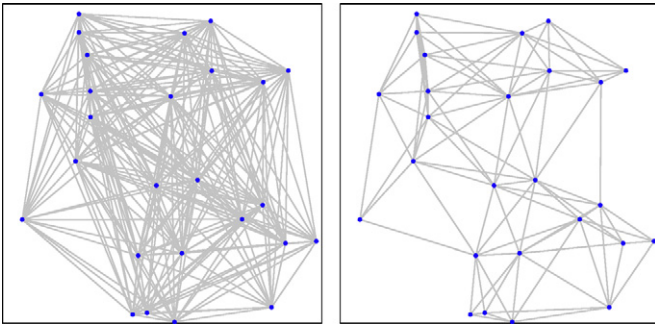


Fig. 1. A topology control protocol trims off inefficient links from a disk graph \mathcal{G} (left) to produce an optimal topology \mathcal{T} (right).

presented. In Section 5, the eligibility criterion for ensuring a fair utilization of energy in wireless sensor networks is discussed. In Section 6, the algorithm for executing the topology control protocol is presented. In Section 7, a brief summary of the mathematical descriptions and algorithms of the protocols that are used for comparisons are presented. In Section 8, the simulation settings and results are discussed. Finally, in Section 9, we provide concluding remarks and future work.

2. Related work

Most existing approaches to topology control apply computational geometry techniques and proximity graphs to build sparse, but connected links.

Timothy et al. (2001), provide a model for computing the most energy-efficient number of hops to relay data from any source in a linear-topology network to a fixed base station. The number of hops depends on a *characteristic distance* and the distance of the source to the base station. The *characteristic distance* itself depends on the propagation environment and radio parameters. We extend this approach to support random deployment in a two-dimensional plain. Jeng and Jan (2007) use Neighborhood Graphs to compute adjustable neighborhood regions and to optimize the node degree. A similar work that optimizes a node degree is proposed in Wattenhofer and Zollinger (2004) – their constructed graph is a subgraph of the Relative Neighborhood Graph (Jaromczyk and Toussaint, 1992) and the protocol uses local information (signal strength information). In both cases, fairness in energy dissipation is not addressed.

Wattenhofer et al. (2001) propose a topology control protocol to dynamically adjust transmission power based on local decisions. Accordingly, a node increases its transmission power until it finds a neighbor node in every direction. But the question how a node trims off inefficient links in case it discovers several neighbors is not addressed.

The topology control protocol of Kung et al. (2008) selects suitable communication nodes, adjusts service loads of critical nodes, and manages sleeping schedules. The protocol principally divides the topology operation into topology formation phase and topology adjustment phases. In the topology formation phase, a link is set up while during the topology adjustment phase, the links are adjusted with an optimal balance of critical nodes in the backbone.

Our strategy is different from the strategies above in the following specific features (Mochaourab and Dargie, 2008):

1. It takes the limitations of a node's hardware (the transceiver) and channel characteristics into account to compute the most optimal communication distance;

2. It aims to optimize the energy consumption of any arbitrary multi-hop link based on local knowledge, i.e., knowledge about neighbors; and,
3. Defines an eligibility metric to ensure that relying nodes are fairly selected. This guarantees a fairly uniform energy consumption throughout the network.

3. Network model

Given a flat topology network¹ of n nodes placed randomly in the Euclidian plane, let \mathcal{V} be the set of vertices representing the nodes and \mathcal{E} be the set of undirected edges representing the communication links between them. The graph of the network is denoted as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. In addition, let $\mathcal{G}_{digraph}$ represent the digraph² of the network with $\mathcal{E}_{digraph}$, the set of undirected edges.

Each node $i, i \in \mathcal{V}$, has a unique identity, id_i , and is represented in the Euclidian plane with its coordinates. A directed edge between two nodes i and j is denoted as $[i \rightarrow j]$, $[i \rightarrow j] \in \mathcal{E}_{digraph}$, and has a distance of $d(i, j)$. An undirected edge between i and j is denoted as $[i \leftrightarrow j]$, $[i \leftrightarrow j] \in \mathcal{E}$. This paper assumes a random distribution of the nodes in a wide rectangular field of deployment. One way to model this type of deployment is by using a two-dimensional Poisson point process (Chao et al., 2008). The points are equally likely to occur anywhere within a bounded region \mathcal{A} , and the probability of finding n nodes in \mathcal{A} is given as:

$$Pr[n \text{ nodes in } \mathcal{A}] = e^{-\lambda} \cdot \frac{(\lambda \cdot \mathcal{A})^n}{n!} \quad (1)$$

where λ is the Poisson process density which is related to the density of the network. The set of neighbors of i , with which i is directly connected are denoted as the set $\mathcal{N}(i)$ and defined as $\mathcal{N}(i) : [i \leftrightarrow j] \in \mathcal{E}_{digraph}$. Let $\mathcal{N}_l(i)$ be the neighbor table list in which the state of each i in $\mathcal{N}(i)$ is stored. $\mathcal{N}_l(i)$ contains the identity, energy reserve, eligibility parameters, and required transmission power to reach each neighbor. Each node has a maximum transmission power of P_{t-max} and can assign varying transmission powers corresponding to each neighboring node. The transmission power from node i to j is denoted as P_{t-ij} . The residual energy of a node i at time t is denoted as e_t^i . Furthermore, all nodes start with equal initial battery capacity \mathcal{E} .

Communication in the network takes place over a wireless medium in which the transmitted signal experiences an attenuation over distance. Moreover, during propagation, the electromagnetic waves experience losses in the form of reflection, diffraction, and scattering. The received signal power, in general, decays as a power law function of the distance separating the transmitter and the receiver. Thus, the received signal power can be written as:

$$P_{rx} \propto \frac{P_{tx}}{d^\gamma} \quad (2)$$

where γ is the path loss exponent and indicates the rate at which the path loss increases with distance. Depending on the presence or absence of a LOS link, different values are assigned to γ .

The power consumption model of the radio transceiver used in this paper is adopted from Timothy et al. (2001); Heinzelman (2000), which considers varying transmission powers to meet minimum receiver sensitivity requirements. This assumption is justified, since most existing transceivers support variable transmission power levels in several discrete steps. An example of such a transceiver is the Texas Instruments Chipcon, CC2420 (Chipcon

¹ In a flat topology network, all nodes play the same roles, both as sensing and as relaying nodes.

² A digraph is a graph with directed edges.

Product, in press). Moreover, the model includes the energy consumed in signal reception which, in today's transceivers, is a considerable amount. A transceiver's energy consumption is mainly accounted for digital signal processing (DSP) and the energy consumed by the front end circuit and the power amplifier/voltage amplifier. The power consumed in transmitting a message at r bits/s over a distance of d meters can be calculated as (Heinzelman, 2000):

$$P_T(d) = (\alpha_{11} + \alpha_2 \cdot d^\gamma)r \quad (3)$$

And the power consumed by the receiver to receive this message is given as (Heinzelman, 2000):

$$P_R = (\alpha_{12}) \cdot r \quad (4)$$

The variables α_{11} and α_{12} are constants and depend on several factors, such as the digital coding and decoding mechanisms; modulation and demodulation, and pulse shaping filters. α_2 depends on the antenna characteristics, channel conditions, amplifier efficiency, and receiver sensitivity.

Two widely used propagation models are the Friss Free Space model ($\gamma=2$) and the Two-Ray Ground propagation model ($\gamma=4$). Depending on the separation distance between the communicating nodes, the propagation model is chosen. A crossover distance which determines this selection is defined in Heinzelman (2000). If the distance is below this crossover distance ($d_{crossover}$), then the free space propagation model is taken, else the Two-Ray Ground propagation model is used. The received signal strength as a function of distance is formulated as (Rappaport, 2001):

$$P_{rx}(d) = \frac{P_{tx}G_tG_r\lambda^2}{(4\pi)^2d^2L} \quad (5)$$

where d is the distance between the transmitter and the receiver in meters; P_{tx} and P_{rx} are the transmitted and received power, respectively; G_t and G_r are the corresponding gains of the transmitting and receiving antenna; h_t and h_r are the height of the transmitting and receiving antenna above ground; λ is the wavelength of the carrier signal; and L is the system loss factor not related to propagation.

Where there is no LOS link between the transmitter and receiver, the Two-Ray Ground model is more accurate than the Friss Free Space model. The received power at a distance d from the transmitter can be expressed as (Rappaport, 2001):

$$P_{rx}(d) = \frac{P_{tx}G_tG_rh_t^2h_r^2}{d^4} \quad (6)$$

The crossover distance is formulated as (Heinzelman, 2000):

$$d_{crossover} = \frac{4\pi\sqrt{L}h_rh_t}{\lambda} \quad (7)$$

4. Fair and efficient topology control

In this section, we establish the basic model of the topology control protocol. The model defines weighted relaying regions in a two-dimensional plane for any arbitrary node in the network. The weighted regions specify the degree of eligibility of a neighboring node to become a relaying node. The eligibility criteria sets a trade-off between minimizing the overall energy cost of a multi-hop communication; and the minimization of disconnected links that occur due to disproportionate energy consumption by individual nodes. The eligibility of each node is computed by taking only local information into account.

4.1. Background

Timothy et al. (2001) sets a theoretical upper bound on the lifetime of a linear-topology wireless sensor network that supports multi-hop communication. Their model calculates the optimal

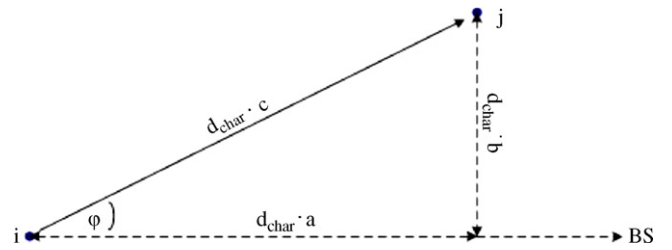


Fig. 2. Hop model.

number of hops based on the notion of a characteristic distance, d_{char} . This distance is computed by taking the hardware components of a transmitter and a receiver as well as the channel's characteristics. Then, for any arbitrary transmitting node, t , a receiving node, r , and a separating distance, D , between them, there exists an optimal number of hops, K_{opt} , such that:

$$K_{opt} = \left\lfloor \frac{D}{d_{char}} \right\rfloor \text{ or } \left\lceil \frac{D}{d_{char}} \right\rceil \quad (8)$$

The characteristic distance, d_{char} , is independent of D and calculated as:

$$d_{char} = \sqrt{[\gamma] \frac{\alpha_1}{\alpha_2(\gamma-1)}} \quad (9)$$

where $\alpha_1 = \alpha_{11} + \alpha_{12}$.

4.2. Hop model

For any arbitrary node i , the position of a neighboring node can be expressed in terms of its deviation from the optimal relaying position. The optimal relaying position is the direct line that connects node i with the base station and it is a function of the characteristic distance, d_{char} . The deviation from this line of a neighbor node is illustrated in Fig. 2.

Taking i as the origin of the coordinate system, the x - and y -coordinates of node i are expressed as $d_{char} \cdot a$ and $d_{char} \cdot b$ respectively, where $a, b \in \mathbb{R}$. The distance from i to j is then $d(i, j) = d_{char} \cdot c$, where $c = \sqrt{a^2 + b^2}$. The x -coordinate is the progress³ of the hop.

Since the computation of a multi-hop link is based on local information only, the optimal number of hops, K_{opt} , can only be an estimation:

$$K = \frac{D}{\bar{a} \cdot d_{char}} = \frac{K_{opt}}{\bar{a}} \quad (10)$$

where \bar{a} is the average value of all a 's.

4.3. Hop efficiency

In order to develop an efficiency measure for a single hop link, we compare the energy consumption of a theoretically optimal multi-hop link with a link that results from our hop model given in the previous section.

The rate at which energy is consumed by a relaying node can be calculated as the overall power consumed during data reception and transmission over a distance d . The Power consumption of a single hop link can be estimated by:

$$P_{relay}(d) = (\alpha_1 + \alpha_2 d^\gamma)r \quad (11)$$

Since the most energy efficient route between node i and the BS is the hop-by-hop LOS link that connects the two nodes, the

³ Progress is the "effective" distance traversed in one hop.

minimum energy rate $P_{link-min}$ that should be consumed during a communication is given as:

$$P_{link-min}(D) = K'_{opt} \cdot P_{relay}(d_{char}) \quad (12)$$

where D is the overall distance to the BS. However, in a randomly deployed sensor network, nodes are not distributed along the optimal link line. Therefore, the power consumed by a link of distance $D' \geq D$ with intervening nodes deviating from the optimal link line can be expressed as:

$$P_{link}(D') = \sum_{i=1}^K P_{relay}(c_i \cdot d_{char}). \quad (13)$$

Taking $P_{link-min}(D)$ as a relative measure, building the ratio of $P_{link-min}(D)$ over $P_{link}(D')$ gives a measure of the efficiency of a chosen link. In maximizing this ratio, the most energy-efficient link can be determined.

$$\frac{P_{link-min}(D)}{P_{link}(D')} = \frac{K'_{opt} \cdot P_{relay}(d_{char})}{\sum_{i=1}^K P_{relay}(c_i \cdot d_{char})} \quad (14)$$

Theorem 1. The overall-link efficiency measure, Λ , of a multi-hop link can be formulated as:

$$\Lambda \leq \frac{\tilde{a} \cdot \gamma}{\bar{c}^\gamma + \gamma - 1} \quad (15)$$

where \bar{c} is the normalized average link distance over d_{char} .

Proof. Defining the overall-link efficiency $\Lambda = (P_{link-min}(D)/P_{link}(D'))$, we can write

$$\begin{aligned} \Lambda &= \frac{K'_{opt} \cdot P_{relay}(d_{char})}{\sum_{i=1}^K P_{relay}(c_i \cdot d_{char})} \\ &= \frac{K'_{opt}(\alpha_1 + \alpha_2 \cdot d_{char}^\gamma)r}{\sum_{i=1}^K (\alpha_1 + \alpha_2(c_i \cdot d_{char})^\gamma)r} \\ &= \frac{K'_{opt}(\alpha_1 + \alpha_2 \cdot d_{char}^\gamma)}{K \cdot \alpha_1 + \alpha_2 \cdot d_{char}^\gamma \sum_{i=1}^K c_i^\gamma} \\ &= \frac{K'_{opt}(\alpha_1 + \alpha_2 \cdot d_{char}^\gamma)}{K \left(\alpha_1 + \alpha_2 \cdot d_{char}^\gamma \cdot \frac{1}{K} \cdot \sum_{i=1}^K c_i^\gamma \right)} \end{aligned}$$

Having c^γ a strictly convex function ($c \in \mathbb{R}^+$, $2 < \gamma < 6$), we can use Jensen's inequality for convex functions, which states that

$$\begin{aligned} \forall \{\lambda_i\}, \lambda_i \in \mathbb{R}^+ \text{ such that } \sum_i \lambda_i = 1 \\ f \left(\sum_i \lambda_i x_i \right) \leq \sum_i \lambda_i f(x_i) \quad (16) \end{aligned}$$

with equality if all x_i s are equal, to get

$$\bar{c}^\gamma \leq \frac{\sum_{i=1}^K (c_i)^\gamma}{K} \quad (17)$$

Using this and the estimated overall-link hop⁴, we can further express:

$$\Lambda \leq \frac{\tilde{a}(\alpha_1 + \alpha_2 \cdot d_{char}^\gamma)}{\alpha_1 + \alpha_2 \cdot d_{char}^\gamma \cdot \bar{c}^\gamma}.$$

Substituting d_{char} given in Eq. (9) in the inequation, we get

$$\begin{aligned} \Lambda &\leq \frac{\tilde{a} \left(\alpha_1 + \alpha_2 \left(\frac{\alpha_1}{\alpha_2(\gamma-1)} \right) \right)}{\alpha_1 + \alpha_2 \left(\frac{\alpha_1}{\alpha_2(\gamma-1)} \right) \bar{c}^\gamma} \\ &\leq \frac{\tilde{a} \left(\alpha_1 + \frac{\alpha_1}{\gamma-1} \right)}{\alpha_1 + \left(\frac{\alpha_1}{\gamma-1} \right) \bar{c}^\gamma} \\ &\leq \frac{\tilde{a} \cdot \left(1 + \frac{1}{\gamma-1} \right)}{1 + \left(\frac{1}{\gamma-1} \right) \bar{c}^\gamma} \\ &\leq \frac{\tilde{a} \cdot \gamma}{\bar{c}^\gamma + \gamma - 1} \quad \square \end{aligned}$$

A transmitting node's knowledge is limited to its immediate neighbors. Therefore, the efficiency model is applied to enable a node compare and select a neighbor that can participate in building a multi-hop link whose overall energy consumption is minimum. **Theorem 1** is employed for the single hop case, substituting the average values with the single hop values. A neighboring node j in the plane of a searching node is Λ_j efficient for the overall link. Hence, its eligibility of being a neighbor of node j is determined accordingly:

$$\Lambda_j = \frac{a \cdot \gamma}{c^\gamma + \gamma - 1} = \frac{\cos \varphi \cdot c \cdot \gamma}{c^\gamma + \gamma - 1} \quad (18)$$

If a transmitting node has no knowledge of the direction of message propagation, i.e., the position of the base station is not known, then it cannot estimate the deviation of a node's position from the optimal link. Hence, φ is set to 0 and Λ can be written as:

$$\Lambda_j = \frac{c \cdot \gamma}{c^\gamma + \gamma - 1} \quad (19)$$

In **Fig. 3**, Λ is plotted when no base station direction information is present at the node. The x -axis is the normalized distance to the neighboring node over d_{char} .

5. Node eligibility metric

The eligibility metric Λ_j derived in the previous section, defines an efficiency measure for a position in the region of transmission range of a node. Thus, a node j within the transmission range of node i acquires this measure Λ_j as its eligibility to be a neighbor. A link established based on this metric ensures an energy efficient multi-hop communication. This is one aspect to consider when building the network, as it is essential to reduce the overall energy dissipation due to routing. Another aspect to consider is fairness between the nodes. In order to ensure that nodes exhaust their energy reserves more uniformly, those nodes that have relatively high energy reserves should be chosen as relying nodes. Hence, we define the metric $\Upsilon_j = (e_j/E)$. Similar to the overall effi-

⁴ The estimated overall-link hop progress normalized over the characteristic distance and denoted as \tilde{a} , is the ratio of hops K'_{opt} , and the number of hops, K : $\tilde{a} = (K'_{opt}/K)$.

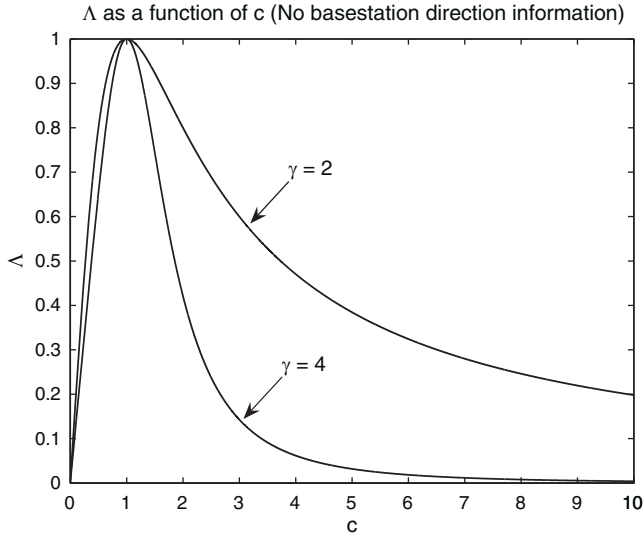


Fig. 3. Plot of Λ for path loss exponents of 2 and 4 where no direction information of the base station exists.

ciency metric, Λ , Υ_j is applied to a neighboring node to measure its relative energy reserve with respect to the other nodes.

Combining both metrics, we can achieve overall-link efficiency and fairness through a common eligibility measure of a neighboring node. Thus, we define:

$$\Psi_j = \Lambda_j \cdot \Upsilon_j \quad (20)$$

A node i having node j in its transmission range calculates Ψ_j , $0 \leq \Psi_j \leq 1$. This determines a measure for node j , for which node i can estimate how eligible it is to be a neighbor.

To accommodate node failure and node mobility, the topology control protocol runs periodically, enabling actual message exchange between the nodes and timely topology adjustment. Moreover, the specific structure of a topology depends on whether nodes have information about the direction of the base station. We denote the graph that is built with the knowledge about the direction of the base station with \mathcal{G}_{FETCD} . Otherwise, it is denoted as \mathcal{G}_{FETC} .

6. Protocol description

Topology formation is accomplished in two phases. The first phase is the neighbor discovery phase in which each node selects k nodes in its neighborhood. The neighbor selection is carried out according to the node eligibility criterion. However, the network graph that is created in this phase is not symmetric. The second phase is concerned with building a symmetric graph from the initial topology that is formed in phase 1. The symmetry is obtained by adding the reverse edge to every asymmetric link.

These phases are described in more detail as follows:

Phase 1: Choosing k Neighboring Nodes (For a generic node i)

1. Node i wakes up at time t_1 , and announces its identity (id_i) and energy reserve ($e_i^{t_1}$) at the maximum power (P_{t-max}).
2. Node i receives the messages from the neighboring nodes and stores their identities in its neighbor list $\mathcal{N}(i)$.
3. Node i estimates the distance to each node in $\mathcal{N}(i)$. Node i has the energy reserves of the neighboring nodes (e_j) as well as the distances to them ($d(i, j)$), where $j \in \mathcal{N}(i)$.
4. Node i calculates Ψ_j , for each neighbor in its list.

5. Node i chooses the k neighbors in its list $\mathcal{N}(i)$ that have the highest value of Ψ . If originally node i has less than k neighbors, then all nodes are chosen.
6. Node i updates its neighbor list according to the chosen nodes in step 5.

The developed graph according to phase 1 of the protocol, has directed links and the graph is a directed graph, $\mathcal{G}_{digraph}$. Hence, a symmetry phase is necessary to enforce symmetry in the graph. In this phase we build the symmetric super-graph of $\mathcal{G}_{digraph}$.

The symmetric super-graph of $\mathcal{G}_{digraph}$ is defined as the undirected graph \mathcal{G} obtained from $\mathcal{G}_{digraph}$ by adding the undirected edge $[i \leftrightarrow j]$ whenever edge $[i \rightarrow j]$ or $[i \leftarrow j]$ is in $\mathcal{G}_{digraph}$. That is, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{E} = \{[i \leftrightarrow j] | [i \rightarrow j] \in \mathcal{E}_{digraph} \text{ or } [i \leftarrow j] \in \mathcal{E}_{digraph}\}$.

Phase 2: Enforcing Graph Symmetry (For generic node i)

1. At time t_2 , node i announces its identity (id_i) and list of Neighbors ($\mathcal{N}(i)$) at maximum power (P_{t-max}).
2. Node i receives the neighbor lists, and calculates the set of symmetric neighbors. Node i checks all neighbor lists and finds if it exists there. When that is the case, it checks if the neighbor list originates from a neighbor in its neighbor list. If not, the corresponding neighbor is added to its list $\mathcal{N}(i)$.

After the symmetric graph is constructed, node i determines for each neighbor in $\mathcal{N}(i)$ the minimum required transmission power to reach it; then this information is stored in its neighbor table list $\mathcal{N}_L(i)$. When communicating with a node in its neighbor list, a node adjusts its transmitter power accordingly. The selected neighbors of a node i are its logical neighbors. That is, there can be nodes in its maximum transmission range it may not be selected as neighbors. These nodes in $\mathcal{N}(i)$ are used for the purpose of routing.

7. Evaluation background

Two types of topology control protocols are considered. The first types exploit knowledge of the geometric properties of nodes and graph theories to establish a network's topology. These types of topology control protocols model networks by considering nodes as points in a Euclidean space and communication links as straight lines that connect two of these points. These types of protocols require little knowledge of the deployment setting, but they perform poorly because they consider the channel characteristics as static. The second types employ probabilistic models to capture and take into account network dynamics. Both types of approaches require an initial graph upon which they apply their algorithms. This same graph can also serve as a reference to evaluate the gains and losses of the topology control protocols.

7.1. Disk graph

Most existing topology control protocols use the disk graph shown in Fig. 1 as a reference – The Disk Graph (DG) has an edge between two nodes u and v if they are at a distance less than d_{max} , $d(u, v) < d_{max}$, where d_{max} refers to the maximum transmission range of a node. Table 1 displays the algorithm we used to generate the Disk Graph Topology.

A network based on the Disk Graph topology will have the highest graph size – which refers to the number of edges (communication links) in the graph. As a result, it is inefficient in terms of its energy consumption. There are different strategies to trim off inefficient links from the DG. For our evaluation, we consider three proximity graphs which are widely referenced in the literature:

Table 1
Algorithm for determining the original topology.

Building the Original Topology $\mathcal{G}_{original}$	
1:	for each $i \in \mathcal{V}$
2:	for each $j \in \mathcal{V}$
3:	if $d(i, j) \leq d_{max}$
4:	$\mathcal{N}(i) \leftarrow j$
5:	$\mathcal{N}_L(i) \leftarrow [i \rightarrow j]$

Table 2
Algorithm for constructing the Relative Neighborhood Graph Topology.

1:	initialize $\mathcal{G}_{rng} \leftarrow \mathcal{G}_{gg}$
2:	for each $i \in \mathcal{V}$
3:	for each $j \in \mathcal{N}(i)$
4:	for each $k \in \mathcal{N}(i)$
5:	if $\max(d(i, k), d(j, k)) < d(i, j)$
6:	remove j from $\mathcal{N}(i)$
7:	remove $[i \rightarrow j]$ from $\mathcal{N}_L(i)$

Table 3
Algorithmic representation for determining the Gabriel Graph Topology.

Building the Gabriel Graph Topology \mathcal{G}_{gg}	
1:	initialize $\mathcal{G}_{gg} \leftarrow \mathcal{G}_{original}$
2:	for each $i \in \mathcal{V}$
3:	for each $j \in \mathcal{N}(i)$
4:	for each $k \in \mathcal{N}(i)$
5:	if $d(i, k)^2 + d(j, k)^2 < d(i, j)^2$
6:	remove j from $\mathcal{N}(i)$
7:	remove $[i \rightarrow j]$ from $\mathcal{N}_L(i)$

relative neighborhood graph, Gabriel graph, and K-neighborhood graph. Proximity graphs have the property of being connected if the original graph (i.e., the Disk Graph) is connected. For the sake of completeness, we briefly summarize these strategies and the algorithms we used to construct the corresponding topologies:

7.2. Relative neighborhood graph

The *Relative Neighborhood Graph* (Jaromczyk and Toussaint, 1992; Santi, 2005) (RNG) of a set \mathcal{V} is a proximity graph such that there exists an edge between points u and v if and only if the lune based region⁵ is empty of other points. The RNG graph has an edge between two points u and v , if there is no other point w such that: $\max\{d(u, w), d(v, w)\} < d(u, v)$ (Table 2).

7.3. Gabriel graph

The *Gabriel Graph* (GG) of a set \mathcal{V} is a proximity graph in which an edge between points u and v exists if and only if a disk whose antipodal points are u and v does not contain any other points in \mathcal{V} (Gross and Yellen, 2004). Mathematically, the GG graph has an edge between two points u and v if and only if there is no other point w such that $d^2(u, w) + d^2(v, w) < d^2(u, v)$. Obviously, $RNG \subseteq GG \subseteq DG$. The GG topology algorithm initializes with the disk graph and the RNG topology algorithm initializes with the GG graph, since the RNG is a subgraph of the GG (Table 3).

7.4. KNeigh

The KNeigh Protocol, as described in Blough et al. (2003), builds the topology based on the k nearest neighbors. The preferred value of k for a large-scale network is derived in the same work and it is set

⁵ A lune is the region of intersection made between two circles that have the same radius.

Table 4
Eligibility metric with and without knowledge of the base station's direction.

Topology	Ψ_j
FETC	$\frac{c_{ij} \gamma_{ij}}{c_{ij}^{\gamma_{ij}} + \gamma_{ij} - 1} \cdot \frac{e_j}{E}$
FETCD	$\frac{\cos \psi_{ij} c_{ij} \gamma_{ij}}{c_{ij}^{\gamma_{ij}} + \gamma_{ij} - 1} \cdot \frac{e_j}{E}$

Table 5
Algorithm for determining the KNeigh Graph Topology.

Determining the KNeigh Topology \mathcal{G}_{KNeigh}	
1:	initialize $\mathcal{G}_{KNeigh} \leftarrow \mathcal{G}_{original}$
2:	for each $i \in \mathcal{V}$
3:	$L(i) \leftarrow []$
4:	for each $j \in \mathcal{N}(i)$
5:	$L(i) \leftarrow d(i, j)$
6:	sort $L(i)$ in ascending order
7:	if number of elements in $L(i) > 9$
8:	i updates $\mathcal{N}(i)$ and $\mathcal{N}_L(i)$ to the first 9 elements in $L(i)$
9:	<i>Symmetry Phase</i>
10:	for each $i \in \mathcal{V}$
11:	for each $j \in \mathcal{N}(i)$
12:	if $[i \rightarrow j] \in \mathcal{N}_L(i)$ and $[j \rightarrow i] \notin \mathcal{N}_L(j)$
13:	remove j from $\mathcal{N}(i)$
14:	remove $[i \rightarrow j]$ from $\mathcal{N}_L(i)$
15:	<i>Pruning Phase</i>
16:	for each $i \in \mathcal{V}$
17:	for each $j \in \mathcal{N}(i)$
18:	if $d(i, j) \leq d_{crossover}$ then $\gamma_{ij} = 2$
19:	else $\gamma_{ij} = 4$
20:	for each $k \in \mathcal{N}(i)$
21:	if $d(i, k) \leq d_{crossover}$ then $\gamma_{ik} = 2$
22:	else $\gamma_{ik} = 4$
23:	if $d(j, k) \leq d_{crossover}$ then $\gamma_{jk} = 2$
24:	else $\gamma_{jk} = 4$
25:	if $d(i, k)^{\gamma_{ik}} + d(j, k)^{\gamma_{jk}} < d(i, j)^{\gamma_{ij}}$
26:	remove j from $\mathcal{N}(i)$
27:	remove $[i \rightarrow j]$ from $\mathcal{N}_L(i)$

to 9. We represent the algorithm for building the KNeigh topology based on the algorithm described in Table 5. The algorithm begins on the original topology, each node selecting its neighbors according to its distances to them. The algorithm has two phases: in the first phase, 9 nearest neighbors are chosen based on their proximity to the node. In the second phase (the symmetry phase that begins at step 10), each node ensures that it is listed as a neighbor node by those nodes which it elects as neighbors. This is to ensure the existence of bidirectional links. If this is not the case, a node drops the neighbors in whose list it is not included. The KNeigh protocol applies further pruning to remove links whose transmission cost in a multi-hop communication is inefficient.

7.5. FETC and FETCD

The algorithm for constructing FETC and its variant, FETCD, is given previously, in Section 4. The two protocols are different in that in FETCD, all nodes have knowledge of the direction of the base station and take this into account to calculate Ψ . Table 4 summarizes how eligibility is measured for the two protocols.

7.6. Routing

The performance of the topology control protocols is best examined when data transmission takes place. For our simulation, we use two different routing protocols: shortest path and energy-aware routing protocols. The first protocol computes a route that has the shortest distance from the source to the destination. The second protocol computes a route that has the maximum overall energy reserves (Table 5).

Table 6
Structure of a node implementation.

Node i				
ID	Position	Energy reserve	Neighbor list	Path to BS
id_i	(x_i, y_i)	e_i	$\mathcal{N}(i)$	$\mathcal{R}(i)$

Table 7
Parameters.

Description	Parameter	Value
Transmitting antenna gain	G_t	1
Receiving antenna gain	G_r	1
Transmitting antenna height	h_t	1.5 m
Receiving antenna height	h_r	1.5 m
Maximum transmitting power	P_{t-max}	0 dBm
Receiver sensitivity	$P_{rx-thresh}$	-85 dBm
Carrier signal wavelength	λ	0.1224 m (2.45 GHz)
System loss factor	L	1
Initial battery capacity	E	
Transmitter electronics energy	α_{11}	26.5 mJ/bit
Receiver electronics energy	α_{12}	59.1 mJ/bit
Efficiency	η_{amp}	0.023
Radio amplifier energy	α_2	$(\gamma = 4)$; $(\gamma = 4)$
Path loss exponent	γ	2 or 4
Relay rate	r	1 bits/s
Characteristic distance	d_{char}	100 m ($\gamma = 2$); 71 m ($\gamma = 4$)

8. Simulation

We use MATLAB® as a simulation tool. We generate the nodes' random placement – a Poisson point process – using a technique described in Lewis and Shedler (1979). Then, with the position information and the channel characteristic parameters, we built the topologies which define for each node a set of neighbors. The parameters that enable a node to make local decisions are displayed in Table 6.

Our simulation is divided into two categories. In the first category, the graphs are investigated from a theoretical point of view, i.e., the graphs' connectivity and node degree are investigated. In the second category, more practical aspects of a wireless sensor network are investigated, namely, the energy cost of the multi-hop links that are built by the different topology control techniques and fairness routing data.

The region of deployment is $500m \times 500m$ two-dimensional plane. The number of nodes deployed in this region is: 100, 200, 300, 400, and 500. Accordingly, different deployment densities are examined. The base station is chosen to be the furthest node with the highest x -coordinate in the deployment. The base station is assumed to have an infinite energy reserve, which is true in reality.

We denote a period of time as a time step in which one bit of information is sent to the base station over a multi-hop link. For each event originating from an arbitrary node i , all eligible relay nodes that are along the path decrease their energy reserve according to our energy model. A relaying node consumes reception power as well as transmission power, depending on its relative distance from a sending node and the next hop. The path loss exponent γ can be either 2 or 4, according to the required transmission distance. Here we introduce the crossover distance, $d_{crossover}$, as in Heinzelman (2000). If the transmission distance is less than $d_{crossover}$, γ is taken as 2. Else, γ is taken as 4. In Table 7, the parameter values used in the simulation are presented.

8.1. Graph connectivity

Connectivity is one of the essential properties of a network graph. The connectivity of a graph is an expression of its 1-connectivity, i.e., each node in the network has at least one multi-hop path that connects it with the base station. For various

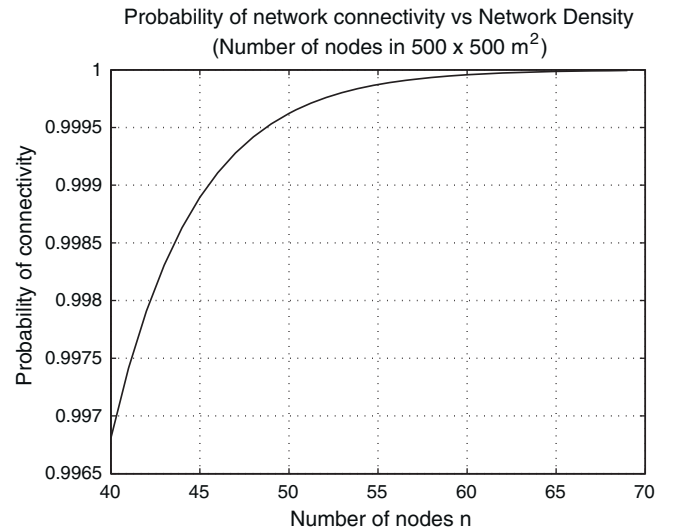


Fig. 4. Probability of graph connectivity.

network densities, we considered different values of k that keep the network connected despite trimming off inefficient links. For a network with a 2D Poisson distribution, the probability that a graph is connected is given as Santi (2005):

$$\Pr(1 - \text{conn}) \approx \left(1 - e^{-\lambda \cdot \pi \cdot d_{max}^2}\right)^n \quad (21)$$

where d_{max} is the maximum transmission range of the nodes; λ is related to the network density; and $n \gg 1$ is the number of nodes. In Fig. 4, we plot the probability of graph connectivity as the density of the network varies from 40 to 70 nodes (worst case situation). As can be seen, deploying less than 70 nodes in an area of $500 \times 500m^2$ does not result in a connectivity probability of 1 regardless of the node degree. In fact, the connectivity probability does not increase for $k > 5$. Comparatively, the GG and RNG graphs can achieve 100% connectivity for the same density. This is one of the reasons why GG and RNG graphs are favored for topology control.

8.2. Energy consumption

We investigated two aspects: the overall network energy consumption and the achievable fairness in the distribution of energy reserve. Fairness is measured by quantifying the variances in the energy reserves of the nodes. We run 100 time-steps for five network densities: 100, 200, 300, 400, and 500. In a time step, 100 nodes are randomly chosen, and one bit of information is sent from each of these nodes along a multi-hop link to the base station. For each network density, 10 random deployments are generated and data transmission (event generation) take place. The average variance of the 10 deployments is used to construct the results for the corresponding network density. This is done for the two routing protocols, namely, the shortest path routing and the energy-aware routing.

First we studied the rate of energy dissipation in the network. This is the amount of energy dissipated in the overall network as a function of the time steps. We normalized the results over the rate of energy consumption of the original topology (the disk graph). Therefore, only relational analysis is displayed.

Fig. 5 displays the energy variations as a function of the time stamps. In both graphs, FETC and FETCD, produce the least energy consumption rate in the network. Moreover, for both cases, the curves reveal that the two protocols have gentler slopes, which indicate lower increasing rates in the variance of energy in the network. The graphs which result in high energy variation are the KNeigh,

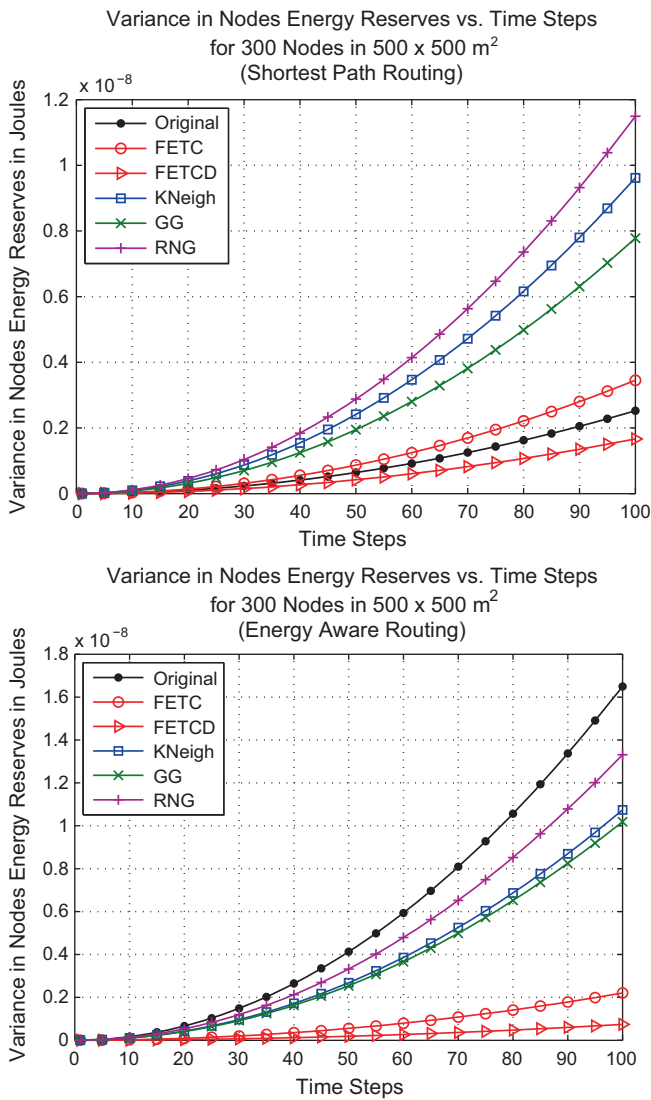


Fig. 5. Variance in the nodes' Energy Reserves.

GG, and RNG topologies. These graphs suffer from the increased number of hops, since they attempt to build the topology based on the nearest neighbors. This increases the energy dissipation of the multi-hop links. The FETC graph has a slight increment in the energy variance as the network density increases. This is expected, as the topology does not exploit knowledge of the direction of the base station to avoid longer routes.

The second aspect of comparison is in the form of normalized energy dissipation as a measure of fairness between nodes. In Fig. 6, the variance of the nodes energy reserves for different node densities after the 100th time step is displayed. The KNeigh, GG, RNG achieve lower fairness compared to the original graph (which has a value of 1).

8.3. Qualitative aspects

So far, the performance of the protocol was discussed quantitatively. In the following subsection, the qualitative aspects are briefly discussed.

8.3.1. Message complexity

The execution of a topology control protocol causes a certain message overhead in the network. The message complexity is

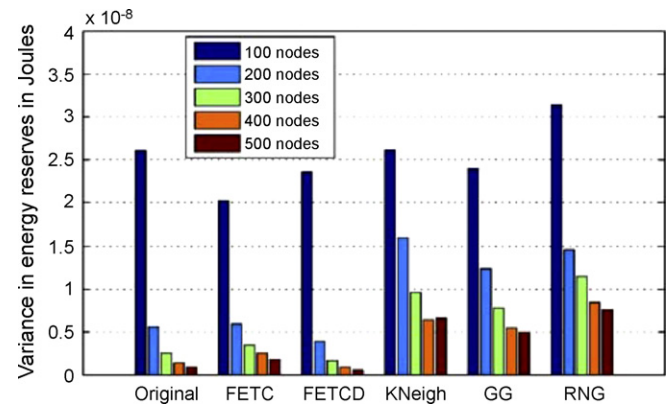


Fig. 6. Variance in the nodes' Energy Reserves after 100 time steps.

an important aspect to mind. Our protocol has a communication complexity⁶ of $\mathcal{O}(n)$. Each node has to send 2 messages in order to determine the topology of the network.

8.3.2. Update policy

To deal with node mobility or failures, the topology control protocol is executed periodically. Finding the optimal rate of computation is not a trivial problem [Brust and Rothkugel \(2007\)](#), and depends on the expected mobility rate. The reconfiguration or re-execution of the topology control protocol can be triggered synchronously or asynchronously. Asynchronous execution is achieved when each node has the choice to determine the time to run the protocol. Synchronous updates, on the other hand, are done when all the nodes execute the topology control protocol at the same time. Our protocol has an asynchronous update policy. Each node initiates the FETC protocol at different times; and has to wait for the replies from the other neighboring nodes for completing the protocol execution. Asynchronous updates are preferred over synchronous updates because of the low retransmission cost at the link layer due to collision.

8.3.3. Node degree

Low node degrees are preferable in order to reduce the overhead of route calculations in the network, but this is often at the cost of the probability of connectivity. A study on the network connectivity and the minimum number of neighbors is carried out in [Xue and Kumar \(2004\)](#). There it has shown that the minimum value of the node degree which guarantees connectivity with high probability is dependent on the number of nodes in the network, such that $\Theta(\log n)$ neighbors are necessary and sufficient for connectivity of the communication graph.

The FETC protocol does not necessarily take the nearest neighbors as criteria for neighbor selection. Instead, it consider logical linking. The case where the logical links and the physical links are identical is when the density of the node distribution is so small that the nearest neighbor distance is greater than the characteristic distance. In that case, the k nearest neighbors are selected as the optimal neighbors. We validate the connectivity issue of our topology for different values of k through intensive simulations; for all the densities we considered, the optimal probability of connectivity can be achieved for $k = 5$.

⁶ The message complexity is defined as the communication effort in terms of the \mathcal{O} -notation that is necessary to run the topology control protocol [Brust and Rothkugel \(2007\)](#).

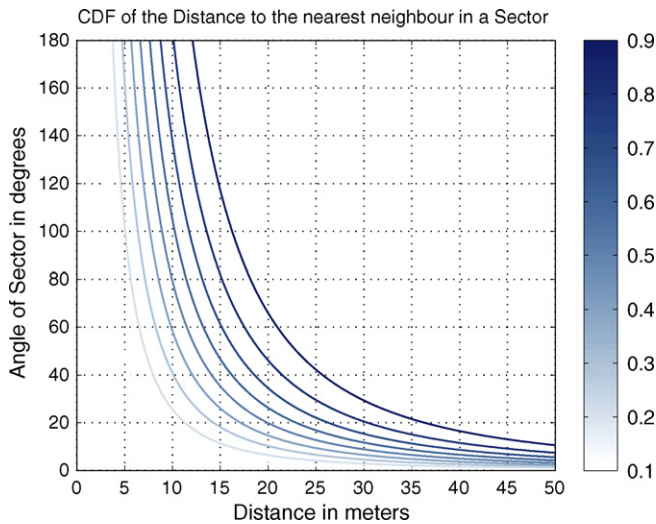


Fig. 7. Contour plot of the distribution function of the distance between a point and its nearest neighbor in a Poisson point process of density $\lambda = 0.01$.

8.3.4. Longer hops

A density-independent distance is proposed to determine the neighboring nodes, which in turn may result in higher transmission powers than other topology control protocols. Transmission over long distance is often argued as being a cause of interference. Since interference is not included in our model, we refer readers to Haenggi (2004) in which convincing reasons in favor of long-hop routings are given.

In Haenggi (2004), it is argued that it is unclear if a single, short duration transmission at high power will bring more interference than multiple short range transmissions. Whereas the former case permits more efficient reuse of the communication channel, the signal to interference ratio (SIR) does not depend on the absolute power levels. Hence, increasing all transmission power levels at the same time does not have a negative impact on any packet reception probability. This indicates that a long-hop transmission does not necessarily cause more interference. Only the signal to interference and noise ratio (SINR) increases.

Topologies that are formed by considering nearest neighbors cannot avoid the presence of nodes that are near to the base station. These nodes exhaust their energy reserve more quickly than others since they are frequently used. This leads to an energy imbalance in the network. Furthermore, such topologies suffer from traffic accumulation at these particular nodes, making them bottle necks for the network information flow. In Dawy and Leelapornchai (2002), the optimal number of relay nodes as a function of the transmission rate is studied. It is shown that as the desired end-to-end rate increases, the optimal number of relay nodes decreases.

Longer hops have higher path efficiencies. The path efficiency is defined as the ration of the Euclidean distance of the end nodes and the multi-hop traveled distance. This is the case, since the probability of finding a relaying node that is near to the optimal line of communication is higher. The distance between a point and its nearest neighbor in a sector ϕ for the Poisson point process is given in Cressie (1993), with the cumulative distribution function given as:

$$F(r) = \Pr(R < r) = 1 - e^{-\lambda \frac{\phi}{2} r^2} \quad (22)$$

where r is the distance to the node and λ is the Poisson process density. In Fig. 7, $F(r)$ is plotted. For small sectors, the estimated distance to the nearest neighbor increases. Hence nodes near to the optimal link line have higher chances to be elected.

9. Conclusion

The energy efficiency of a wireless sensor network and the lifetime maximization problem is tackled by considering two aspects. The overall network energy consumption efficiency and fairness. Based on theoretical work on upper bounds of the network lifetime, we exploited the notion of a characteristic distance, d_{char} , that is dependent on the radio characteristic and the channel condition. From a node's view point, an estimation is made over the neighboring nodes on their overall-link efficiency in relaying a message. This is done according to their positions relative to an optimal relaying position and the position of the base station. The efficiency estimation is made hop by hop. Fairness in energy utilization, and thereby connectivity, is addressed by taking the energy reserves of the nodes in the neighbor selection criteria into account.

The simulation results confirmed that our topology is not as sparse as the RNG, GG, and KNeigh topologies. However, with respect to the original topology (the mesh topology), the node degree is slightly increased with network density. Interesting results are obtained concerning the energy dissipation rate in the overall network. Unlike the other topology control protocols, the energy dissipation rates are little affected by increasing network densities. Moreover, concerning the energy reserves between the nodes, contrary to the RNG, GG, and KNeigh topologies, we have minimized imbalance. The results showed that nearest neighbor topologies are energy inefficient for high density networks. The original topology (disk graph), on the other hand, contains inefficient long links which significantly decreased the energy efficiency of the network. These results show that our network topology suits to prolong the lifetime of the network.

References

- Blough, D.M., Leoncini, M., Resta, G., Santi, P., 2003. The k-neigh protocol for symmetric topology control in ad hoc networks. In: *MobiHoc '03: Proceedings of the 4th ACM International Symposium on Mobile ad hoc Networking & Computing*. ACM, New York, NY, USA, pp. 141–152.
- Brust, M.R., Rothkugel, S., 2007. A taxonomic approach to topology control in ad hoc and wireless networks. In: *ICN '07: Proceedings of the 6th International Conference on Networking*. IEEE Computer Society, Washington, DC, USA, p. 25.
- Chao, X., Dargie, W., Lin, G., 2008. Energy model for H2S monitoring wireless sensor network. In: *CSE '08: Proceedings of the 2008 11th IEEE International Conference on Computational Science and Engineering*. IEEE Computer Society, Washington, DC, USA, pp. 402–409.
- Chipcon Product data sheet: <http://www.chipcon.com/>.
- Cressie, N., 1993. *Statistics for Spatial Data*. Wiley Series in Probability and Statistics.
- Dawy, Z., Leelapornchai, P., 2002. Optimal number of relay nodes in wireless ad hoc networks with non-cooperative accessing schemes. In: *ISITA 2002*.
- Ephremides, A., 2002. Energy concerns in wireless networks. *IEEE Journal of Wireless Communication*, 48–59.
- Gross, J.L., Yellen, J., 2004. *Handbook of Graph Theory*. CRC Press.
- Haenggi, M., 2004. Twelve reasons not to route over many short hops. In: *Proceedings of the 60th IEEE Vehicular Technology Conference*, pp. 3130–3134.
- Heinzelman, W.B., 2000. *Application-specific protocol architectures for wireless networks*. PhD Thesis. (Supervisor-Chandrasekaran, Anantha P. and Supervisor-Balakrishnan, Hari).
- Jaromczyk, J.W., Toussaint, G.T., 1992. Relative neighborhood graphs and their relatives. *Proc. IEEE* 80 (9), 1502–1517.
- Jeng, A.A.-K., Jan, R.-H., 2007. The r-neighborhood graph: an adjustable structure for topology control in wireless ad hoc networks. *IEEE Trans. Parallel Distrib. Syst.* 18 (4), 536–549.
- Kung, H.-Y., Huang, C.-M., Ku, H.-H., Tung, Y.-J., 2008. Load sharing topology control protocol for harsh environments in wireless sensor networks. In: *AINA '08: Proceedings of the 22nd International Conference on Advanced Information Networking and Applications*. IEEE Computer Society, Washington, DC, USA, pp. 525–530.
- Lewis, P.A.W., Shedler, G.S., 1979. Simulation of nonhomogeneous Poisson processes by thinning. *Nav. Res. Logist. Quart.* 26, 403–414.
- Mochaourab, R., Dargie, W., 2008. A fair and energy-efficient topology control protocol for wireless sensor networks. In: *Proceedings of the 2nd ACM International Conference on Context-Awareness For Self-Managing Systems*. CASEMANS '08, vol. 281, Sydney, Australia, May 19–19, 2008. ACM, New York, NY, pp. 6–15.

- Rappaport, T., 2001. *Wireless Communications: Principles and Practice*. Prentice Hall PTR, Upper Saddle River, NJ, USA.
- Santi, P., 2005. Topology control in wireless ad hoc and sensor networks. *ACM Comput. Surv.* 37 (2), 164–194.
- Timothy, M.B., Garnett, T., Ch, A.P., 2001. Upper bounds on the lifetime of sensor networks. In: *ICC 2001*, pp. 785–790.
- Wattenhofer, R., Li, L., Bahl, P., Min Wang, Y., 2001. Distributed topology control for power efficient operation in multi-hop wireless ad hoc networks. In: *12th Annual Joint Conference of the IEEE Computer and Communications Societies, Infocom*, pp. 1388–1397.
- Wattenhofer, R., Zollinger, A., 2004. XTC: A practical topology control algorithm for ad-hoc networks. *Parallel and Distributed Processing Symposium, International* 13, 216a.
- Xue, F., Kumar, P.R., 2004. The number of neighbors needed for connectivity of wireless networks. *Wirel. Netw.* 10 (2), 169–181.