Dynamic and Efficient Brokering of Energy Suppliers and Consumers in a Smart Grid

Waltenegus Dargie
Chair for Computer Networks
Technical University of Dresden
01062 Dresden, Germany
Email:waltenegus.dargie@tu-dresden.de

Omar Alfandi
College of Technological Innovation
Zayed University
P.O. Box 144534 Abu Dhabi, U.A.E
Email:Omar.AlFandi@zu.ac.ae

Mohamad Badra
College of Technological Innovation
Zayed University (Dubai Campus)
P.O. Box 144534 Abu Dhabi, U.A.E
Email:mohamad.badra@zu.ac.ae

***Abstract*—One of the fundamental tasks of a smart-grid is achieving an optimal balance between the supplied and consumed energy in the grid. The optimal balance avoids underutilisation as well as overloading of energy sources; minimises the cost of energy transportation and storage; and reduces the price of energy. In this paper we propose a stochastic model for associating energy-suppliers with consumers having matching characteristics in a probabilistic sense. The optimal number of users a particular supplier can serve is described in terms of the probability density functions of its energy production and the demand of consumers. We shall demonstrate both analytically and numerically that an optimal balance can be achieved when the supplied energy, the demand for energy, and the number of users associated with a particular supplier, all, have a normally distributed probability distribution function (pdf).**

***Index Terms*—Smart grid, supply-demand model, probabilistic model, optimal number of consumers, probability distribution function**

I. Introduction

The worldwide energy consumption and energy price are increasing steady [1], [2]; the former affected by factors such as population growth, improved quality of life in middle-income economies, increment in the number of electrically operated equipments and devices per household, the growing influence of ICT in the world, and similar emerging factors; and the former due to the inherent scarcity of resources. The concomitant effect of a growing energy demand is the negative impact of energy related wastes on the environment. These collective developments have necessitated the inclusion or the consideration of alternatives both at the energy generation and energy distribution stages.

At the energy generation stage, alternative as well as complementary energy sources, such as wind and solar, are presently being integrated in different countries to provide additional but also environment-friendly (clean) energy to consumers. As of July 2015, the statistics from EWEA reveals that in "the first six months of 2015, Europe fully grid connected 584 commercial offshore wind turbines, with a combined capacity totalling 2,342.9 MW. Overall, 15 commercial wind farms were under construction. Once completed, these wind farms will have a total capacity of over 4,268.5 MW."[1] At the distribution stage, the main aim is to ensure the efficient but also the flexible utilisation of energy by consumers.

The traditional grid structure (producer-controlled and centrally managed) is optimised for the seamless integration of energy producers, so that the availability of energy sources can be abstracted from customers, so that the fluctuation of energy supply can be avoided as energy sources join and leave the grid. In other words, the relationship between energy suppliers and consumers is loosely coupled. The structure, however, performs poorly when it comes to participating consumers, because the communication is unidirectional. Consequently, there is a strong drive to distribute energy supply and for a smart integration of consumers in the grid, so that they can cooperate with utility companies as to when, for how long, and how much energy they should draw from the grid in specific circumstances. For example, consumers can be enticed by incentives (in the form of extra Kilowatt-hours or lowered price per kilowatt-hours) whenever they refrain from overloading the grid during peak hours [3].

The vision of a smart grid is to achieve both the seamless integration of energy suppliers and participatory energy consumption. Different energy suppliers expose their product to retailers through the grid. The retailers or utility companies buy, aggregate, temporarily store, and distribute energy. The price they set on the energy they sale depends on many factors such as the relative stability of the energy sources and the cost of storage and distribution. Since the smart grid includes a communication and smart metering facilities, utility companies can also directly negotiate with consumers about the energy consumption modalities, so that consumers can avoid both the underutilisation and the excess utilisation (overloading) of the system, both of which are sources of inefficiency; the former producing high storage cost whilst the latter producing undesirable fluctuation of energy distribution and potential damage to the infrastructure.

One of the advantages of the smart grid concept is the ease with which local (small-scale) and ubiquitous energy

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[1] http://www.ewea.org/statistics/ (Last accessed on December 02, 2015).

producers can find customers within their proximity. For example, in rural areas, small towns, and villages, small-scale energy producers can sale their product to local customers in the same way small-scale farmers can sale dairy and milk products to their local customers thereby significantly reducing storage and transportation costs. Customers also profit from a local strategy because they can have access and contribute to environment-friendly and affordable products and play a vital role in strengthening the local economy. In this paper we mathematically determine the number of customers that can be associated with a specific energy producer so that the energy produced by the producer can be efficiently consumed (1) with a minimum demand for energy storage and (2) without considerably overloading the producer. Our mathematical approach models both the produced and the consumed energy as random variables and estimate peak energy production and consumption in a probabilistic sense.

The remaining part of the paper is organised as follows: In Section II we discuss related work and demonstrate how our work complements state-of-the-art. In Section III we present our mathematical approach in detail. In Section IV we evaluate our approach numerically. Finally, in Section V we provide concluding remarks.

II. Related Work

Several approaches have been proposed to manage the smart grid and to involve customers in the decision-making process, so that energy can be consumed in an efficient and affordable manner. Some of these strategies involve (1) avoidance of excessive fluctuation of energy supply as energy sources join and leave the grid [4]; (2) dynamic pricing based on the availability and the demand of energy and the avoidance of both underutilisation and excessive utilisation of energy; (3) participatory decision-making in order to avoid excessive energy draw during peak times; and (4) dynamic scheduling of energy utilisation to ensure a steady and balanced energy consumption. Except (1), the aim of which is improving the reliability of the grid, all the others are some form of demand side management (DSM) [5], [6], [7] the core concept of which is introduced by the Electric Power Research Institute (EPRI) in the 1980s [8]. The core idea of DSM is to persuade users to modify or postpone their demand during peak hours of consumption. This way a balance in the grid between energy supply and energy utilisation can be maintained. In order to make customers apt for persuasion, utility companies offer them some form of incentives.

One of the aspects of DSM is the integration of smart meters [9] and a communication infrastructure into the smart grid system. Among its merits, this idea enables to gather sufficient statistics about consumption behaviours and timing and to communicate the statistics with the utility companies, so that based on the statistics, customer profiling, load shifting and control [10], [5], and dynamic pricing [11] can be made. Alternatively, the customers themselves can manage their energy load voluntarily [7], [12] by predicting load and price distribution and by avoiding the use of electricity during peak hours [13].

Different strategies and mathematical models have been proposed to support DSM. In [14], [15], [16] a stochastic optimisation model is proposed to determine the optimal energy load in a smart grid and the corresponding optimal energy price. In [11] a game-theoretic strategy is proposed to control energy demand at the user side. The idea is to install the optimal strategy at the customer side so that customers can identify the most suitable and, therefore, the most affordable price and schedule their consumption accordingly. The approach attempts to distribute the time of energy utilisation. Similarly, in [17] an autonomous and distributed demand-side management system is proposed. In [18], a four-stage Stackelberg game strategy is employed where energy suppliers are grouped into two distinct price groups. In the first group, unreliable but cheap suppliers can be found whereas in the other group reliable but costly suppliers are grouped. The strategy aims to determine the optimal supplier combination for a customer, so that its demand is met for an optimal price.

Most of the proposed approaches attempt to influence users' behaviour in managing the smart grid. But in most realistic situations user' behaviour cannot easily be influenced because they are causally connected to different factors related to their lifestyle, work and familial situations, and convenience. In this paper we propose an optimal pairing strategy that takes production and consumption statistics into consideration. Hence, our approach, instead of influencing suppliers or consumers behaviours, aims to identify matching statistics and the optimal number of consumers a given supplier can satisfy. The strategy also minimises the cost of storage because energy can be utilised as soon as it is produced. With a localisation algorithm, our strategy can be most suitable for small-scale suppliers and consumers in villages, small towns, and remote places where small-scale wind, solar, and similar renewable energy can be pervasively produced.

III. Concept

Neither the energy produced nor the energy consumed in a smart grid is a deterministic quantity. This is particularly true for renewable energy the amount of which depends on the wind or solar that can be harvested at any given time from any given location, which is never deterministic. Consequently, we model the amount of energy that can be produced by a specific producer and the energy consumed by a specific customer as random variables. A random variable is sufficiently explained by its probability density function (pdf), which assigns a probability term to any real value assignable to the random variable.

Suppose the energy produced by the i-th supplier is $\mathbf{s}_i$ and the energy consumed by the j-th consumer is $\mathbf{c}_j$ kWh. The pdf of $\mathbf{s}_i$ and $\mathbf{c}_j$ are $f_i(s)$ are $f_j(c)$, respectively. Generally, a single supplier produces more energy than can be consumed by a single consumer. In small communities, categorising consumers based on their function is plausible. For example, consumers can be categorised as households, hotels, restaurants, etc. The classification enables to assume that members of a group have similar energy consumption characteristics. In other words, they have similar density functions.

The overall energy consumed by $\mathbf{n}$ consumers (where $\mathbf{n}$, too, is a random variable, as the number of consumers drawing energy from a grid at any given time cannot be known in advance) can be expressed as:

$$\mathbf{c} = \sum_{i=1}^{\mathbf{n}} \mathbf{c}_i \tag{1}$$

where $\mathbf{n}$ and $\mathbf{c}_i$ are statistically independent for all i, since the number of users accessing the grid and the amount of energy consumed by individual consumers have nothing to do with one another. Notice that $\mathbf{c}$ is a random variable as it is a result of the summation of multiple random variables.

In order to understand how the pdf of $\mathbf{c}$ is affected by the pdf of the $\mathbf{c}_i$, suppose we have only two consumers and each consumer consumes at any given time either 8 kWh or 9 kWh with equal probabilities. So the overall energy drawn from the grid can be one of the following combinations: $[(8+8),(8+9),(9+8),(9+9)]$. In terms of probabilities, the amount of energy that can be drawn by the two consumers simultaneously has the following distribution: $[16(0.25), 17(0.5), 18(0.25)]$. Similarly, if the number of distinct amounts of energy increases for the two consumers, the distribution of the overall energy consumption changes significantly. For example, if they consume $[7,8,9,10]$ kWh with equal probabilities (0.25) at any time, the energy drawn from the grid will have six different values: $[14,15,16,17,18,19,20]$, obviously, some values are more probable than others (16 kWh being the most probable). In general, if the pdfs of the two random variables are continuous, the pdf of $\mathbf{c}$ can be determined as the convolution of the pdfs of $\mathbf{c}_1$ and $\mathbf{c}_2$. The pdf of $\mathbf{c}$ becomes normal or Gaussian if more than two $\mathbf{c}_i$ are added (following the central limit theorem), which is the case for our case, because we assumed that potentially a large number of consumers draw energy from the grid. Figure 1 shows how the pdf of $\mathbf{c}$ evolves to become a normal distribution as the the number of $\mathbf{c}_i$ increases from 2 to 3.

Hence, the aggregate energy demand of $\mathbf{n}$ customers is a normally distributed random variable with mean η_n and variance σ_n^2. If we know the mean (η) and variance (σ^2) of the demand of each customer, then it is possible to adequately describe the overall demand as follows:

$$\begin{aligned} \eta_n =& E\left\{\mathbf{x}_1 + \mathbf{x}_2 + ... + \mathbf{x}_n\right\} \\ =& E\left\{\mathbf{x}_1\right\} + E\left\{\mathbf{x}_2\right\} + ... + E\left\{\mathbf{x}_n\right\} \\ =& \eta_1 + \eta_2 + ... + \eta_n = n\eta \end{aligned} \tag{2}$$

Note that in Equation 2 we assumed that the expected consumption of the n customers is the same. Likewise, the variance of the aggregate consumption can be calculated as follows:

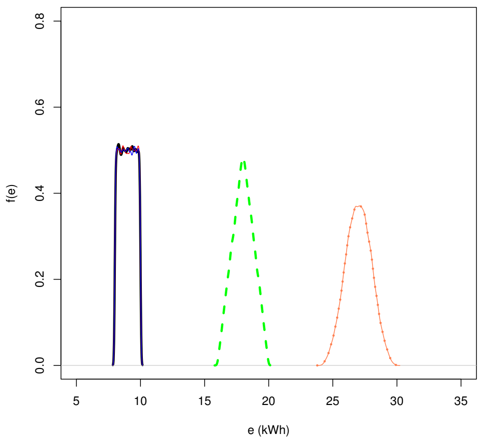

Figure 1: In accordance with the central limit theorem, the pdf (density function) of the sum of multiple random variables tends to be normal (gaussian) regardless of the nature of the pdf of the individual random variables. From left to right: The pdfs of three individual random variables (uniform), the pdf of $\mathbf{c}_1 + \mathbf{c}_2$ and the pdf of $\mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3$

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$$\sigma_n^2 = E\left\{(\mathbf{c} - \eta_n)^2\right\} = E\left\{\mathbf{c}^2\right\} - \left(E\left\{\mathbf{c}\right\}\right)^2 \tag{3}$$

To demonstrate the solution of Equation 3, assume that we have only two customers: $\mathbf{c}_1$ and $\mathbf{c}_2$. Thus, $\sigma_{n=2}^2$ is expressed as:

$$\begin{aligned} \sigma_{n=2}^2 =& E\left\{(\mathbf{c}_1 + \mathbf{c}_2)^2\right\} - (2\eta)^2 \\ =& E\left\{\mathbf{c}_1^2\right\} + E\left\{\mathbf{c}_2^2\right\} + 2E\left\{\mathbf{c}_1\mathbf{c}_2\right\} - 4\eta^2 \end{aligned} \tag{4}$$

Each $\mathbf{c}_i^2$ can be expressed in terms of its variance and mean; since $\sigma^2 = E\left\{\mathbf{c}_i^2\right\} - \eta^2$, $E\left[\mathbf{c}_i^2\right] = \sigma^2 + \eta^2$. Hence,

$$\sigma_{n=2}^2 = \left(\sigma^2 + \eta^2\right) + \left(\sigma^2 + \eta^2\right) + 2\eta^2 - 4\eta^2 = 2\sigma^2 \tag{5}$$

Note that in Equation 3 we have assumed that the two consumers are independent and $2E\left\{\mathbf{c}_1\mathbf{c}_2\right\} = 2\eta\eta = 2\eta^2$. In general, for n consumers, Equation 4 becomes:

$$\sigma_n^2 = n\sigma^2 \tag{6}$$

The number of customers drawing energy from the grid simultaneously at any give time of the day cannot be known in a deterministic sense and, therefore, should be considered as a random variable, as we already mentioned above. This also means that the expected amount of energy that can be drawn from the grid by these $\mathbf{n}$ customers (Equation 2) and the variance (Equation 6) are both random variables:

$$\eta_{\mathbf{n}} = \mathbf{n}\eta \tag{7}$$

And,

$$\sigma_{\mathbf{n}}^2 = \mathbf{n}\sigma^2 \tag{8}$$

where $\mathbf{n}$ denotes that the number of consumers is modelled as a random variable. Considering $\mathbf{n}$ and $\mathbf{c}$ are statistically independent and assuming that the pdf of $\mathbf{n}$ or the expected number of customers that can draw energy from the grid simultaneously is known, we can determine the expected mean and variance of the energy that can be drawn from the grid simultaneously by the group of customers as follows:

$$E\left\{\eta_{\mathbf{n}}\right\} = \eta_n \eta \tag{9}$$

where η_n is the expected number of customers that draw energy from the grid simultaneously. And,

$$E\left\{\sigma_{\mathbf{n}}^2\right\} = \eta_n \sigma^2 \tag{10}$$

In conclusion, the aggregate demand (consumption) of the $\mathbf{n}$ customers will have a normal distribution according to the central limit theorem and as shown in Figure 1. The expected mean and variance of this pdf are given by Equations 9 and 10, respectively. From the supplier side, energy is efficiently utilised if it can be consumed as soon as it is produced, so as to eliminate the cost of energy storage. For this to happen, the expected amount of energy that can be produced by the j-th supplier should be matched by the appropriate amount of demand:

$$E\left\{\mathbf{s}_j\right\} = \eta_n \eta \tag{11}$$

Consequently, for a specific group of consumers with a known η (i.e., households, restaurant, hotels, pastry, etc.), the optimal expected number of customers that can be served by the j-th supplier can be expressed as:

$$\eta_n = \frac{E\left\{\mathbf{s}_j\right\}}{\eta_n \eta} \tag{12}$$

IV. Evaluation

We used the *R statistical tool* to validate our model. The number of users that consume the energy produced by a single supplier in a smart grid is set to be normally distributed, with the expected number of consumers being 20. The variance of the distribution is 0.5. Each household consumes on average 9.6 kWh per day. This is based on the latest statistics available for Germany (the average electricity consumption per electrified household)[2]. According to the central limit theorem, the aggregate energy utilisation of the $\mathbf{n}$ consumers has a normally distributed pdf with mean and variance specified by Equations 2 and 6, respectively.

If a supplier, the energy supply of which has a normally distributed pdf with mean and variance equalling the mean and variance described by Equations 9 and 10 is found, then the optimal balance between energy generation and consumption will be attained with a minimum amount of surplus or deficiency of energy. Fig. 2 shows this condition. In the figure, the dashed lines indicate the demand of customers

[2]https://www.wec-indicators.enerdata.eu/household-electricity-use.html (last accessed on December 08, 2015).

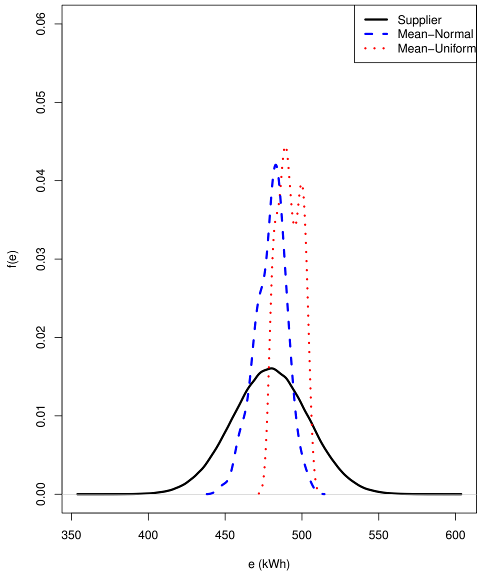

Figure 2: The difference between the pdf of the supplier and the average energy demand of $\mathbf{n}$ consumers when $\mathbf{n}$ is normally distributed (blue) and uniformly distributed (red) with a mean number of users being 20 for both cases.

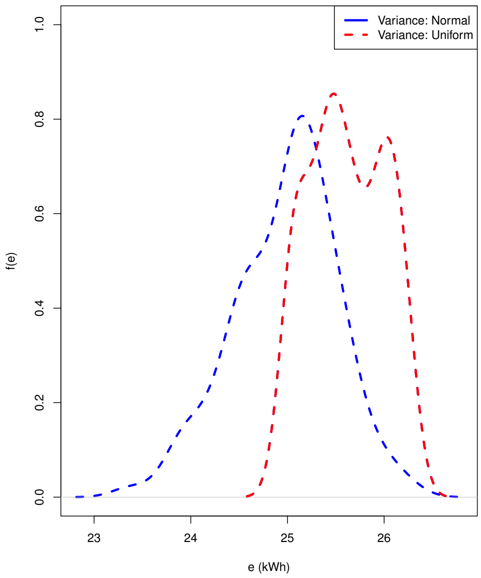

Figure 3: The variance of the energy utilised by the $\mathbf{n}$ number of users which indicate the expected mismatch between the energy supplied by the j-th supplier and the energy consumed by the $\mathbf{n}$ customers. The blue line indicates the fluctuation in the energy demand when $\mathbf{n}$ is normally distributed and the red line indicates the fluctuation in the energy demand when $\mathbf{n}$ is uniformly distributed.

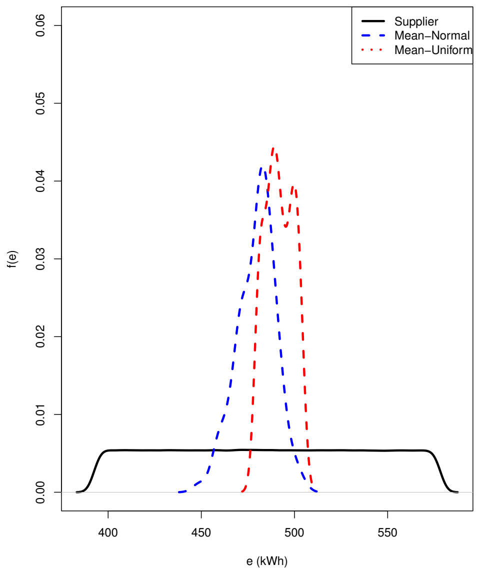

Figure 4: The difference between the pdf of the supplier and the average energy demand of **n** consumers when **n** is normally distributed (blue) and uniformly distributed (red) with a mean number of users being 20 for both cases.

when the energy consumption of each household is normally distributed (blue) and uniformly distributed (red). In both cases, the aggregate energy consumption of the **n** consumers is normally distributed with a slightly different characteristics. The solid line (black) shows the distribution of the energy generated by the supplier. As can be seen, the expected energy demand and the expected energy supply overlap, confirming that the approach is optimal. Fig. 3 shows the discrepancy between the energy supplied and consumed: a value above the mean indicating energy surplus and a value below the mean indicating a shortage of energy. The expected discrepancy is $(25/475) \times 100\% = 5.26\%$ when the energy utilisation of each household is normally distributed. The number slightly increases when it is uniformly distributed (5.47%).

In contrast, in Fig. 4 the supplier produces a uniformly distributed energy. Even though the **mean**, **min** and **max** of the supplied and demanded energy are the same, there is, however, a significant difference between the supplied and demanded energy, leading to either a significant surplus of energy (requiring storage) or shortage of energy.

V. Conclusion

In this paper we proposed a probabilistic model for coupling small-scale suppliers of energy with small-scale energy consumers. The model is useful for villages, small towns, and remote locations where private households can find small-scale energy suppliers in their vicinity. The energy that can be generated by renewable sources, the energy consumed by a single household, and the number of consumers that can draw energy simultaneously at any given time cannot be known in a deterministic sense. Consequently, we modelled them as random variables and specified their statistical characteristics with probability distribution functions (pdf). Based on the statistical features and making use of the central limit theorem, we proposed the optimal number of users that can be associated with a single supplier. The matching is optimal when all the three random variables have normally distributed pdfs.

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